69.

<u>Problem 17.17 (HR)</u>

A cube floating on mercury has one-fourth of its volume submerged. If enough water is added to cover the cube, what fraction of its volume will remain immersed in mercury? Does the answer depend on the shape of the body?

<u>Solution:</u>

Let the volume of the cube be V. As the volume of mercury displaced by the cube is $V/4$, the mass of the cube will be

$$M = \rho_{Hg} V/4.$$

Let V' be the volume of the cube that is immersed in mercury and $V - V'$ be the volume of water that is displaced when water is added to mercury such that the cube is covered with water. In that situation mass of mercury and mass of water displaced by the cube will be

$$M' = V' \rho_{Hg}$$

and

$$M'' = \left(V - V'\right) \rho_{water} \ .$$

As $M = M' + M''$, we have the relation

$$\rho_{Hg} V/4 = V' \rho_{Hg} + \left(V - V'\right) \rho_{water} \ ,$$

or

$$\frac{V'}{V} = \frac{\left(\rho_{Hg} / 4 - \rho_{water}\right)}{\left(\rho_{Hg} - \rho_{water}\right)}.$$

As

$\rho_{Hg} = 13.6$ g cm^{-3},

and

$\rho_{water} = 1$ g cm^{-3},

we find

$$\frac{V'}{V} = 0.19 \ .$$

Answer does not depend on the shape of the body.