68.

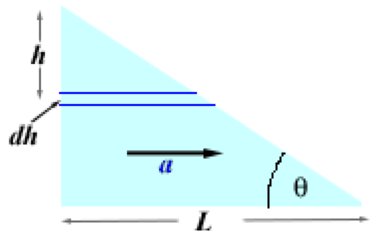
Problem 17.21(HR)

Consider the horizontal acceleration of a mass of liquid in an open tank. Acceleration of this kind causes the liquid surface to drop at the front and to rise at the rear. (a) *We have to show that the liquid surface slopes at and angle θ with the horizontal, where* $\tan\theta = a/g$*, a being the horizontal acceleration.* (b) *How does the pressure with depth?*

Solution:

Concepts that we will use in solving this problem are that pressure force is perpendicular to fluid surface and that hydrostatic gauge pressure varies with depth by the standard relation $p = h\rho g$. If a fluid element is under acceleration *a* then the net force on it due to gauge pressure has to be equal to its mass times the acceleration *a*.

We consider an element of the fluid that is moving horizontally with acceleration. Relevant data concerning the motion of this element have been shown in the following diagram.



Let L be the length and W be the width of this element of fluid, and let θ be the angle that its surface makes with the horizontal when it is moving horizontally with acceleration a. Let the atmospheric pressure acting on this element of fluid be p_0.

Horizontal forces acting on this element are the force due to pressure acting on its vertical face, height $L\tan\theta$ and width W, and the horizontal component of the force due to atmospheric pressure p_0 acting on the surface of length $L/\cos\theta$ and width W. Force on the vertical face can be calculated by integrating

$$\Delta F = \left(p_0 + h\rho g\right)Wdh$$

from 0 to $L\tan\theta$. We get

$$F_v = \int_0^{L\tan\theta} \left(p_0 + h\rho g\right)Wdx$$

$$= Wp_0 L\tan\theta + W\rho g\left(L\tan\theta\right)^2/2 \ .$$

Horizontal component of the force due to atmospheric pressure p_0 on the slanting surface will be

$F_s = -Wp_0 L \tan\theta.$

The net horizontal force on the fluid element will be

$F_v + F_s$, that is

$F = W\rho g\left(L\tan\theta\right)^2 / 2.$

Mass of the fluid element is

$M = WL^2 \tan\theta\rho / 2.$

As it is moving with acceleration a, from the Newton's second law of motion we have

$WL^2 \tan\theta\rho a/2 = W\rho g\left(L\tan\theta\right)^2 / 2,$

or

$a = g\tan\theta.$