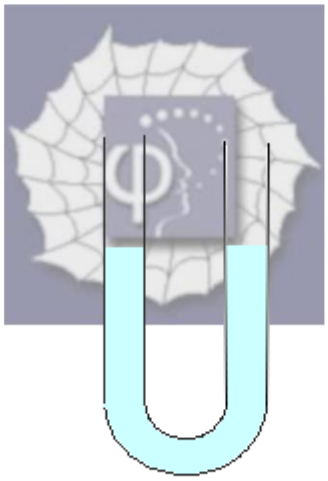
67.

Problem 17.7 (HR)

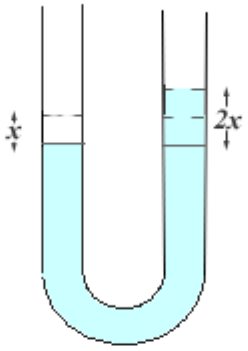
A U-tube is filled with a single homogeneous liquid. The liquid is temporarily depressed in one side by a piston. The piston is removed and the level of the liquid in each side oscillates. We have to show that the period of oscillation is $\pi\sqrt{2L/g}$ *where L is the total length of the liquid in the tube.*



U-tube filled with homogeneous liquid

Solution:

A U-tube is filled with a single homogeneous liquid. At equilibrium liquid levels on both sides of the U-tube will be equal. When the level of the liquid say in the left-side of the U-tube has been depressed by *x* units it will be lifted up by *x* units in the other side of the U-tube.



Level in the left-side is depressed by x units

The liquid column will experience a downward force $2x\rho gA$, where A is the cross-sectional area of the U-tube, because of gravity or equivalently due to the pressure difference. Mass of the liquid column is $AL\rho$, as L is the total length of the liquid column. Motion of the liquid column can be considered in terms of the motion of its centre of mass (cm). Equation of motion of the cm of the liquid column is

$$(\rho AL)\frac{d^2x}{dt^2} = -2A\rho gx,$$

or

$$\frac{d^2x}{dt^2} + \frac{2g}{L}x = 0.$$

It is an oscillatory motion with angular frequency

$$\omega = \sqrt{\frac{2g}{L}}.$$

Period of oscillation T is related to ω as

$$T = \frac{2\pi}{\omega}.$$

Therefore, the period of oscillation of the liquid column is $T = \frac{2\pi}{\sqrt{\frac{2g}{L}}} = \pi\sqrt{\frac{2L}{g}}.$