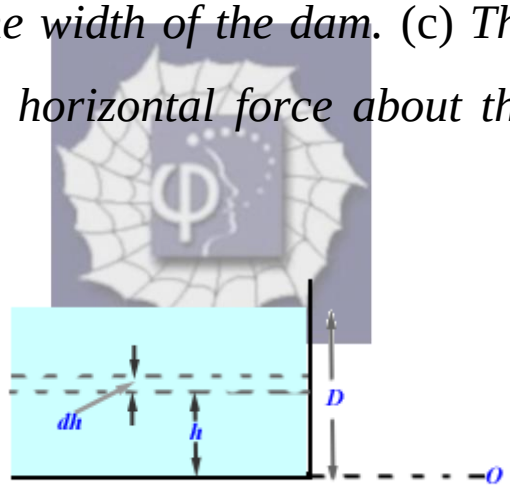
66.

Problem 15.23P (HRW)

Water stands at a depth D behind the vertical upstream face of a dam. Let W be the width of the dam. We have to find (a) *the resultant horizontal force exerted on the dam by the gauge pressure of the water and* (b) *the net torque owing to that gauge pressure about a line O parallel to the width of the dam.* (c) *The moment arm of the resultant horizontal force about the line through O.*



Solution:

Width of the dam is *W* m. Gauge pressure at depth *D-h* from the water level at the top of the dam is $\rho(D-h)g$.

Let us consider a strip *dh* of width *W* for computing the force on the dam due to water at depth *D-h* from the

water level at the top of the dam. Area of this strip is Wdh. Force ΔF is

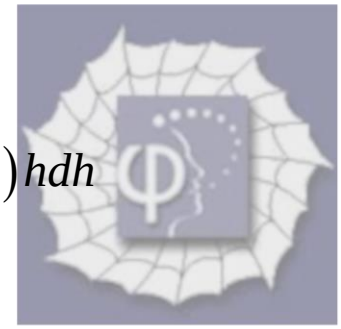
$$\Delta F = \rho g\left(D-h\right)Wdh.$$

The force on the dam due to gauge pressure of water will be

$$F = \int_0^D \rho gW\left(D-h\right)dh$$

$$= \rho gWD^2/2.$$

Torque about a line through O, parallel to the width of the dam, will be



$$\tau = \int_0^D \rho gW\left(D-h\right)hdh$$

$$= \frac{\rho gWD^3}{6}.$$

Moment arm, *L*, of the resultant horizontal force about the line through *O* will, therefore, be given by the relation

$$FL = \tau.$$

We have worked out the expressions for *F* and τ, which give

$$L = \frac{W\rho g D^3/6}{W\rho g D^2/2}$$
$$= \frac{D}{3}.$$