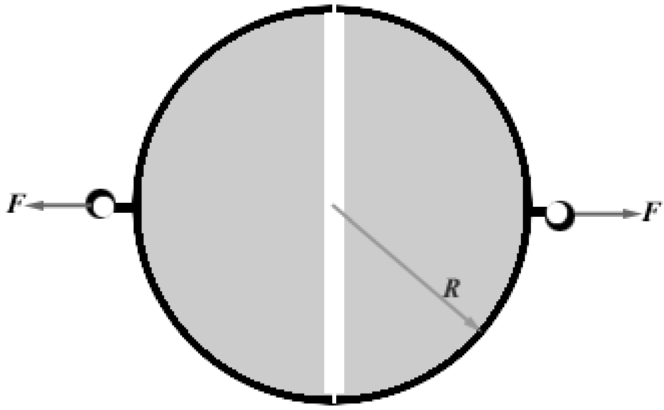
61.

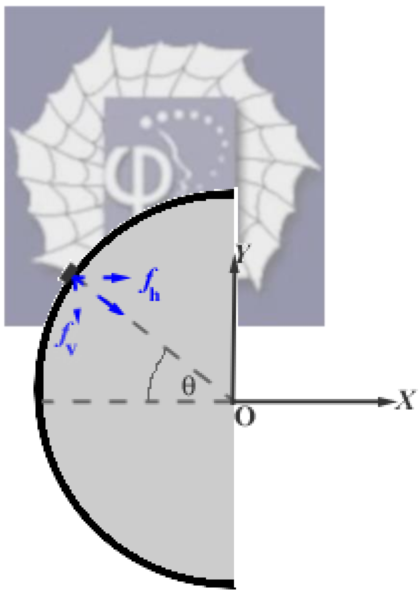
Problem 15.8P (HRW)

In 1654 Otto Von Guerick, inventor of the air pump, gave a demonstration before the noblemen of the Holy Roman Empire in which two teams of eight horses could not pull apart two evacuated brass hemispheres. (a) *Assuming that hemispheres have thin walls, so that R of the spherical shell may be considered both the inside and the outside radius, we have to show that the force required to pull apart the hemisphere is* $F = \pi R^2 \Delta p$, *where* Δp *is the difference between the outside and inside the sphere.* (b) *Taking* R=1.0 ft *and inside pressure as* 0.10 atm, *we have to find the force the teams of horses would have had to exert to pull apart the hemispheres.* (c) *Why are two teams of horses used? Would not one team have proved the point just as well if the hemispheres were attached to a sturdy wall?*



Solution:

(a)



Let us first work out the force on the left-hemisphere assuming that the difference in pressure outside and inside the sphere after it has been evacuated is Δp. Let us consider an infinitesimal element of area tangential to the surface at longitudinal angle θ and azimuthal angle ϕ,

$$\Delta a = R^2 \sin\theta d\theta d\phi.$$

Force due to difference in pressure acting on the infinitesimal element of area Δa is

$$f = \Delta p\, R^2 \sin\theta d\theta d\phi.$$

As shown in the figure, its direction is radial. Its x-component that is along the axis *OX* is

$$f_h = \Delta p\, R^2 \sin\theta d\theta d\phi \cos\theta.$$

We note that the components of the forces in the vertical direction in the top and the bottom quadrants, which is along the axis OY, will cancel out in pairs and the net force on the hemisphere due to difference in pressure will be in the horizontal direction. The net force on the sphere can be calculated by integrating f_h on the left hemisphere. That is

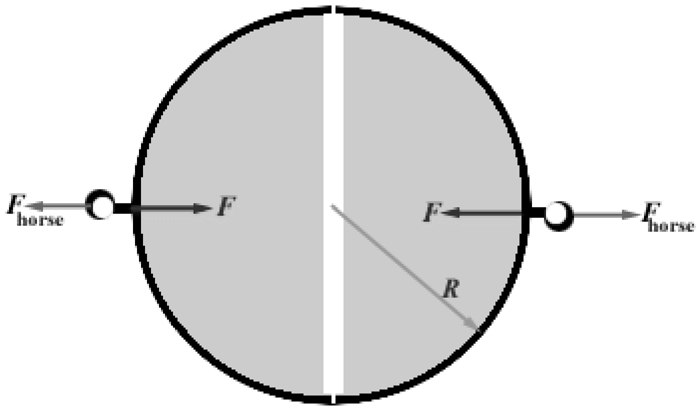
$$F = \int_0^{\pi/2} \sin\theta \cos\theta d\theta \int_0^{2\pi} \Delta p R^2 d\phi$$

$$= 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta$$

$$= \pi R^2 \Delta p.$$

(b)

Therefore, in the *Otto Von Guerick's* experiment the net force on each hemisphere was F and their directions were as shown in the figure.



R is the radius of each hemisphere. Therefore, forces to be exerted by horses on each side of the hemisphere for pulling apart the hemisphere were

$F_{horse} = F = \pi R^2 \Delta p$.

From the data given in the problem, we have

R=1.0 ft, $\Delta p = \left(1.0 - 0.10 = 0.90\right)$ atm.

As, 1 atm $= 14.7$ lb/ in^2,

$$F_{horse} = F = \pi R^2 \Delta p = \pi \left(12\right)^2 \times 0.90 \times 14.7 \text{ lb}$$
$$= 5985 \text{ lb.}$$

(c)

If instead of using two pairs of horses the sphere was attached to a sturdy wall and force applied by one set of horses, the experiment would have only tested that the molecular forces are able to keep the two hemispheres joined together under difference in outside and inside pressures.