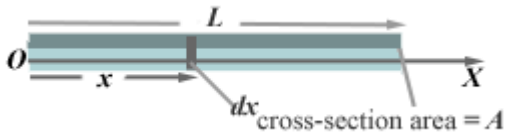
58.

Problem 14.47 (RHK)

A rotor blade 5.27 m *long is composed of material of density* 4.55 g cm^{-3} *and ultimate tensile strength* 446 MN m^2. *We have to calculate its greatest rotational speed. We can assume that the blade rotates about an axis perpendicular to and through one end of the blade.*

Solution:

Let the cross-sectional area of the blade be A m^2. Length of the rotor blade $L = 5.27$ m. Let ω be the angular speed of the rotor blade.



The centripetal force on the blade can be calculated by integrating contribution of an infinitesimal piece of the blade of width dx at a distance x from the end of the blade connected to the rotor over the length of the blade, $\rho A\omega^2 x\,dx$. That is

$$F = \int_0^L \rho A \omega^2 x \, dx$$
$$= \tfrac{1}{2} \rho A \omega^2 L^2 \ .$$

Tensile stress on the blade is $\dfrac{F}{A} = \dfrac{\rho \omega^2 L^2}{2}$.

As the ultimate tensile strength is 446 M N m^{-2}, we have the relation

$$\frac{\rho \omega^2 L^2}{2} = 446 \times 10^6 \text{ N m}^{-2},$$

or,

$$\omega = \sqrt{\frac{2 \times 446 \times 10^6}{4.55 \times 10^3 \times 5.27^2}} \text{ rad s}^{-1}$$
$$= 84 \text{ rad s}^{-1} = \frac{84 \times 60}{2\pi} \text{ rev min}^{-1}$$
$$= 802 \text{ rev min}^{-1} \ .$$

Therefore, the greatest possible rotation speed of the blade is 802 rev min^{-1}.