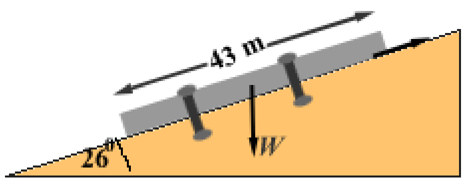
57.

<u>Problem 13.53P (HRW)</u>

A rectangular slab of slate rests on an 26^0 *incline. The slab has dimensions* 43 m *long,* 2.5 m *thick and* 12 m *wide. Its density is* $3.2\,\mathrm{g\,cm^{-3}}$. *The coefficient of static friction between the slab and the underlying rock is* 0.39. (a) *Calculate the component of the slab's weight acting parallel to the incline.* (b) *Calculate the static frictional force on the slab. By comparing* (a) *and (*b), *we can see that the slab is in the danger of sliding and is prevented from doing so only by chance protrusions between the slab and the underlying rock.* (c) *To stabilise the slab, bolts are driven perpendicular to the incline. If each bolt has a cross-sectional are of* $6.4\ \mathrm{cm^2}$ *and will snap under a shearing stress of* $3.6\times10^8\ \mathrm{N\,m^{-2}}$, *what is the minimum numbers of bolts needed? We can assume that the bolts do not affect the normal force.*



Solution:

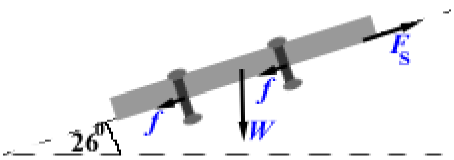
Volume of the slab, $V = 43 \times 2.5 \times 12\ \text{m}^3 = 1290\ \text{m}^3$.

Density of slate, $\rho = 3.2\ \text{g cm}^{-3} = 3.2 \times 10^3\ \text{kg m}^{-3}$.

Weight of the slab,

$W = V\rho g = 1290 \times 3.2 \times 9.8 \times 10^3\ \text{N} = 4.05 \times 10^7\ \text{N}$.

Free-body diagram of the slab is



(a)

Component of slab's weight acting parallel to the

incline$= W \sin 26^0 = 4.05 \times 10^7 \times .438\ \text{N} = 1.77 \times 10^7\ \text{N}$.

(b)

Maximum force of static friction,

$F_S = W \cos 26^0 \times \mu = 4.05 \times 10^7 \times 0.898 \times 0.39\ \text{N}$

$= 1.42 \times 10^7\ \text{N}$.

By comparing (a) and (b), we note that the maximum force of static friction is less than the component of the weight parallel to the incline.

(c)

Each bolt can withstand a force

$f = \text{shearing stress} \times a = 3.6 \times 10^8 \times 6.4 \times 10^{-4}\ \text{N} = 2.3 \times 10^5\ \text{N}.$

Therefore, the number of bolts needed for holding the

slab on the incline $= \dfrac{(1.77 - 1.42) \times 10^7}{2.3 \times 10^5} = 15.2$.

Therefore, the minimum number of bolts required for holding the slab on the incline will be 16.