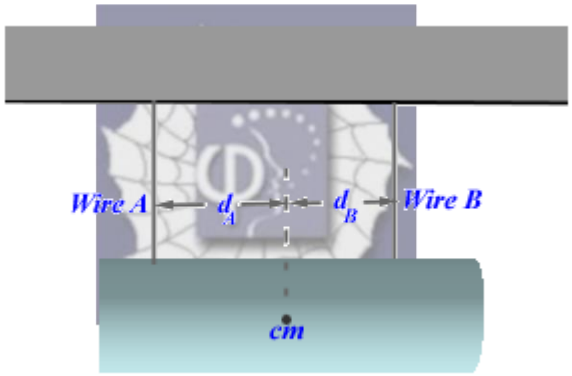
56.

Problem 13.55P (HRW)

A 103 kg *uniform log hangs by two steel wires, A and B, both of radius* 1.20 mm. *Initially, wire A was* 2.50 m *long and* 2.00 mm *shorter than wire B. The log is now horizontal. What forces are exerted on it by* (a) *wire A and* (b) *wire B?* (c) *What is the ratio* d_A/d_B *?*



Solution:

Let *W* be the weight of the log. Let the initial lengths of the wires *A* and *B* be l_A and l_B, respectively. Let $f_A W$ be the pull on wire *A* and $f_B W$ be the pull on the wire *B* due to the weight of the log. From the data of the problem we note the relations

$$f_A d_A = f_B d_B,$$

and

$f_A + f_B = 1.$

Let Δl_A be the increase in length of wire *A* caused by the stress on it and let Δl_B be the corresponding increase in length of wire *B*. As the log becomes horizontal because of the increase in lengths of wires *A* and *B*, we have

$\Delta l_A = \Delta l_B + 2 \times 10^{-3}$ m.

In the following we will approximate

$l_A \approx l_B = l = 2.50$ m.

Young's modulus of steel $E = 200 \times 10^9$ N m^{-2}. Let area of cross-section of wires *A* and *B* be

$a = \pi \times 1.2^2 \times 10^{-6}$ m^2 $= 4.52 \times 10^{-6}$ m^2.

From Hook's law of elasticity, we have

$$\Delta l = \frac{Fl}{Ea}.$$

We, therefore, have

$$\frac{f_A W l}{Ea} - \frac{f_B W l}{Ea} = 2.00 \times 10^{-3} \text{ m},$$

or

$$f_A - f_B = \frac{2.00 \times 10^{-3} \times 200 \times 10^9 \times 4.52 \times 10^{-6}}{103 \times 9.8 \times 2.5} = 0.716 .$$

We thus find

$f_A = 0.858$

and

$f_B = 0.142.$

(c)

And,

$$\frac{d_A}{d_B} = \frac{f_B}{f_A} = 0.166.$$

(a) and (b)

Force exerted on wire *A* is

$$f_A W = 0.858 \times 103 \times 9.8 \text{ N} = 866 \text{ N},$$

and

$$f_B W = 0.142 \times 103 \times 9.8 \text{ N} = 143 \text{ N}.$$