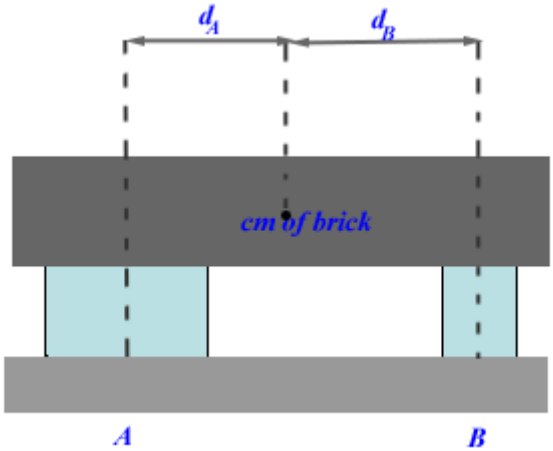
55.

Problem 13.54P (HRW)

A lead brick rests horizontally on cylinders A *and* B. *The areas of the top faces of cylinders are related by* $A_A = 2A_B$*; the Young's moduli of cylinders are related by* $E_A = 2E_B$. *The cylinders had identical lengths before the brick was placed on them. What fraction of the brick's weight is supported* (a) *by cylinder A and* (b) *by cylinder B? The horizontal distances between the centre of mass of the brick and the centres of lines of the cylinders are* d_A *for cylinder A and* d_B *for cylinder B.* (c) What is the ratio d_A/d_B?



Solution:

(a) and (b)

Let W be the weight of the brick. Let f_A and f_B be the fractions of the weight W supported by cylinders A and B, respectively. As d_A and d_B are the distances from the centre of mass of the brick where $f_A W$ and $f_B W$ are acting on the cylinders A and B. This implies

$$f_A d_A = f_B d_B.$$

By definition

$$f_A + f_B = 1.$$

As the cylinders had identical lengths before stress was applied and undergo same change in length after being subjected to stress by the weight of the brick, this requirement is equivalent to the condition that strains on the two cylinders are the same. This gives the relation

$$\frac{f_A W}{A_A E_A} = \frac{f_B W}{A_B E_B}.$$

Using the data given in the problem, the above relation can be rewritten as

$$f_A = 4 f_B.$$

Therefore,

$$f_A = \tfrac{4}{5} \text{ and } f_B = \tfrac{1}{5}.$$

(c)

Using the results above, we find

$$\frac{d_A}{d_B} = \frac{1}{4}.$$