## 52.

## Problem 13.45P (HRW)

A car on a horizontal road makes an emergency stop by applying the brakes so that all four wheels lock and skid along the road. The coefficient of kinetic friction between tyres and road is 0.40 . The separation between the front and rear axles is 4.2 m , and the centre of mass of the car is located 1.8 m behind the front axle and 0.75 $m$ above the road. The car weighs 11 kN . We have to calculate (a) the braking deceleration of the car, (b) the normal force on each wheel, and (c) the braking force on each wheel.


## Solution:

(a)

The coefficient of kinetic friction between tyres and road, $\mu_{k}$, is 0.40 . Weight of the car is 11 kN . Let $N_{f}$ be the normal force on the front tyres and $N_{r}$ be the normal force on the rear tyres. The sum of the normal forces on the front and rear tyres will be equal to the weight of the car. That is

$$
2 N_{f}+2 N_{r}=11 \mathrm{kN} .
$$

When brakes are applied all the wheels gets locked and car skids with decelerating force, $F_{b}$, equal to the force of kinetic friction. As force of kinetic friction is equal to the normal force times the coefficient of kinetic friction, we have

$$
F_{b}=0.40 \times 11 \mathrm{kN}=4.4 \mathrm{kN} .
$$

Mass of the car M is equal to its weight divided by the acceleration due to gravity,
$M=\frac{11 \times 10^{3}}{9.8} \mathrm{~kg}=1122 \mathrm{~kg}$.

Therefore, decelerating acceleration of the car when the brakes have been applied will be $a=\frac{4.4 \times 10^{3}}{1122} \mathrm{~m} \mathrm{~s}^{-2}=3.92 \mathrm{~m} \mathrm{~s}^{-2}$.
(b)

When brakes have been applied car is not in translational equilibrium, but as it is not toppling it certainly is in rotational equilibrium. We will calculate the torque about the centre of mass of the car and for equilibrium the net torque has to be zero. For calculating the torque we will first draw the free-body diagram of the car. It is as shown below.


We have drawn the $X$ and $Y$ axes in the plane of the diagram. $Z$-axis is perpendicular to $X$ and $Y$ axes and is
coming out of the plane. We have fixed the origin of the co-ordinate system at the centre of mass of the car. The $z$-component of the torque is defined as $\tau_{z}=x F_{y}-y F_{x}$.

The total $\tau_{z}$ about the centre of mass is
$\tau_{z}=2(-1.8) N_{f}-2(-0.75) \mu N_{f}+2(2.4) N_{r}-2 \times(-0.75) \mu N_{r}=0$, or,
$5.4 N_{r}-3.0 N_{f}=0$,
$N_{f}=1.8 N_{r}$.

As, $2 N_{f}+2 N_{r}=11 \mathrm{kN}$, we have
$5.6 N_{r}=11 \times 10^{3} \mathrm{~N}$,
or,

$$
N_{r}=1964 \mathrm{~N},
$$

and
$N_{f}=3535 \mathrm{~N}$.

## (c)

Braking force on front wheels is

$$
\mu N_{f}=0.4 \times 3535 \mathrm{~N}=1414
$$

and the braking force on rear wheels is $\mu N_{r}=0.4 \times 1964 \mathrm{~N}=786 \mathrm{~N}$.

