51.

**Problem 13.24P (HRW)**

A balance is made up of a rigid, massless rod, supported at a point not at the centre of the rod and is free to rotate about it. It is balanced by unequal weights placed in the pans at each end of the rod. When an unknown mass m is placed in the left-hand pan, it is balanced by a mass m_1 placed in the right-hand pane; and when the mass m is placed in the right-hand pan, it is balanced by a mass m_2 in the left-hand pan. We have to show that $m = \sqrt{m_1 m_2}$.

Solution:

Free-body diagram of the balance in the first situation is as shown.
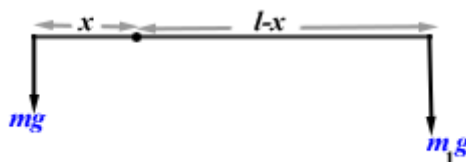
As the balance is in equilibrium, the torque about the point of suspension has to be zero. If the point of
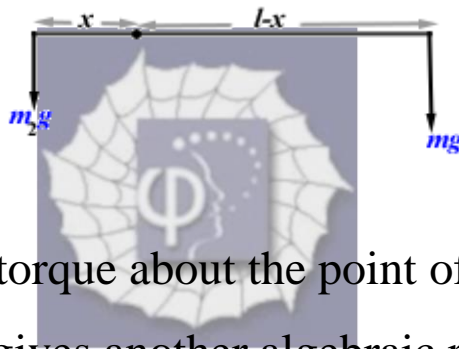
suspension is at a length x from the left-hand end of the balance that is of length l, condition of equilibrium give the algebraic equation

$$mgx = m_1 g (l - x),$$

or,

$$\frac{x}{l - x} = \frac{m_1}{m}.$$

Free-body diagram of the balance in the second situation is as shown below.

For equilibrium torque about the point of suspension has to be zero. This gives another algebraic relation

$$m_2 gx = mg (l - x),$$

or,

$$\frac{x}{l - x} = \frac{m}{m_2}.$$

From these algebraic equations, we get the relation

$$\frac{m_1}{m} = \frac{m}{m_2},$$

or,

$$m = \sqrt{m_1 m_2} \ .$$