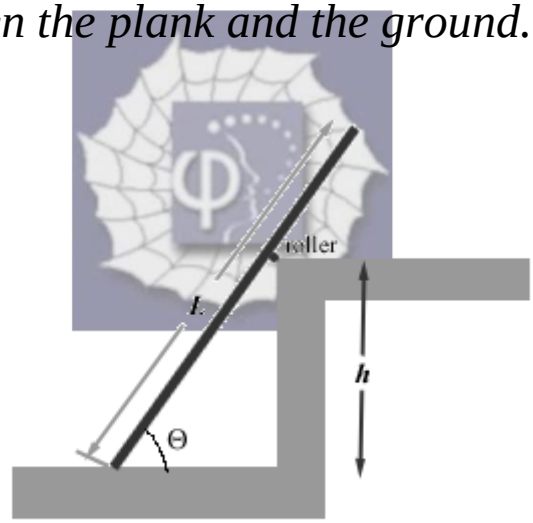
50.

Problem 13.40P (HRW)

A uniform plank, with length L=20 ft *and weight* W=100 lb, *rests on the ground and against a frictionless roller at the top of a wall of height* h = 10 ft. *The plank remains in equilibrium for any value of* $\theta \geq 70^0$ *but slips if* $\theta < 70^0$. *We have to find the coefficient of static friction between the plank and the ground.*

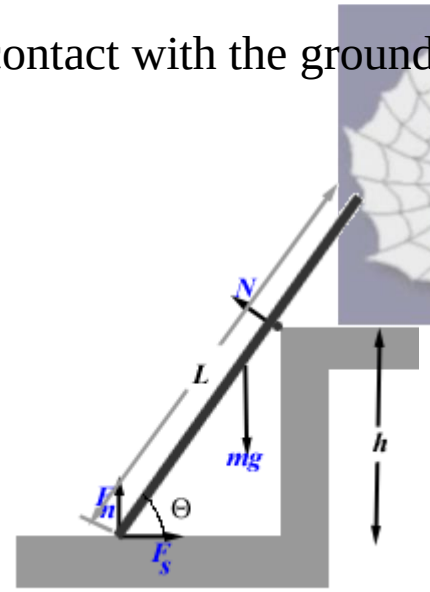


Solution:

We will consider the free-body diagram of the plank. As the roller is frictionless, it exerts a normal force N on the plank at the point of contact. At the contact point of the plank with the ground there will be a horizontal force F_s due to friction, which resists the sliding back of the

plank, and normal force F_n acting in the vertical direction.

In the problem it has been given that when $\theta = 70^0$, the maximum force F_s that can arise will be μF_n, where μ is the coefficient of static friction between the plank and the ground. Conditions of equilibrium are that the vector sum of the external forces has to be zero and torque due to external forces about any point has to be zero. We will compute the torque about the point where the plank is in contact with the ground. We get the following algebraic relations because of these conditions.



$F_n + N\cos\theta = mg,$

$F_s = N\sin\theta$. As $F_s = \mu F_n,$ we get

$\mu F_n = N\sin\theta.$

$$N \times \frac{10}{\sin\theta} = mg \times 10\cos\theta,$$

or

$$N = mg\sin\theta\cos\theta.$$

From these equations we can solve for μ. We get

$$\mu = \frac{\sin^2\theta\cos\theta}{1-\sin\theta\cos^2\theta}.$$

In this relation on substituting

$\sin 70^0 = 0.94,$ and

$\cos 70^0 = 0.34,$ we find

$\mu = 0.34.$