47.

Problem 13.31P (HRW)

Four identical uniform bricks, each of length L, are put on top of one another in such a way that part of each extends beyond the one beneath. Find, in terms of L, the minimum value of (a) a_1, (b) a_2, (c) a_3, (d) a_4, *and* (e) h, *such that the static is in equilibrium.*

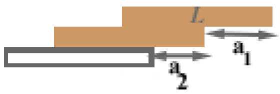
Solution:

Key to answering this problem is that for equilibrium and for requiring the condition of minimum value of displacements a_1 , a_2 , a_3 , a_4 , the centre of mass of each block of bricks has to be on top of the edge of the brick on which the block is resting. We will answer this problem in steps.

We have been given that the bricks are identical and each brick has length L.

The centre of mass of the topmost brick, l_1, has to be in its middle. Therefore, $a_1 = L/2$.

We next consider the top two bricks as a block resting on the edge of the third brick, such that the block does not topple. This will happen when the centre of mass of the top two bricks lies on top of the edge of the third brick.



We will calculate the centre of mass of the top two bricks, l_2, measuring horizontal distance from the left-hand edge of the third brick.

$$l_2 = \frac{(a_2 + \tfrac{1}{2}L)m + (a_1 + a_2 + \tfrac{1}{2}L)m}{2m} = L,$$

or,

$$\frac{2a_2 + \tfrac{3}{2}L}{2} = L,$$

or,

$$a_2 = \tfrac{1}{4}L.$$

Making similar calculations as above, we can easily find

$a_3 = \tfrac{1}{6}L$, and $a_4 = \tfrac{1}{8}L$.

We thus find

$$a_1 + a_2 + a_3 + a_4 = \frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{8} = \frac{25}{24} L.$$