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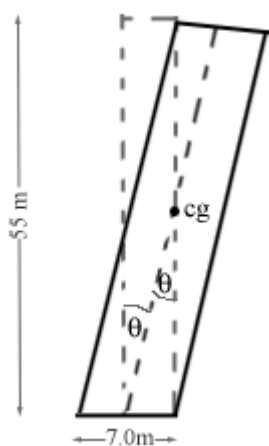
Problem 13.3E (HRW)

The leaning tower of Pisa is 55 m high and 7.0 m in diameter. The top of the tower is displaced 4.5 m from the vertical. We can treat the tower as a uniform, circular cylinder. We have to answer the following:

(a) *What additional displacement, measured at the top, will bring the tower to the verge of toppling?*

(b) *What angle with the vertical will the tower make at that moment?*

Solution:



The tower of Pisa will reach the verge of toppling down when its lean has reached the stage that its centre of gravity (cg) shifts vertically above its edge (as shown in the figure). At that stage a slight additional lean will

result in a torque in clockwise direction caused by the weight of the tower acting at its centre of gravity, which is the same as the centre of mass (cm) of the tower. As the height of the tower is 55 m, its cm will be at its

midpoint, which is 27.5 m from the base. Angle of lean θ when the tower is at the verge of toppling down is given by

$$\sin \theta = \frac{3.5}{27.5} = 0.127,$$

or,

$$\theta = \sin^{-1} 0.127 = 7.31^\circ = 0.127 \text{ rad} .$$

At that stage the top of the tower would have shifted from the vertical position by $s = 55 \times 0.127 \text{ m} = 7.01 \text{ m}$. Because of the lean the top of the tower is already displaced by 4.5 m from the vertical position, therefore, an additional lean of 2.5 m would bring the tower to the verge of toppling down.