36.

Problem 12.33 (RHK)

A stick 1.27 m *long is held vertically with one end on the floor and is then allowed to fall. We have to find the speed of the other end when it hits the floor, assuming that the end on the floor does not slip.*

Solution:

Let *m* be the mass of the stick. Its length *l* is 1.27 m. When the stick is standing vertically up, its energy is the potential energy in the gravitational field of the Earth. As the stick is uniform, its centre of gravity will coincide with its centre of mass. Therefore, when the stick is in the vertical position, the potential energy of the stick is more relative to when it has fallen and is lying on the floor by the amount

$$E = \Delta PE = mg\,l/2.$$

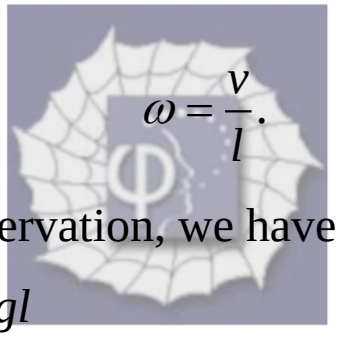
Rotational inertia of the stick with respect to perpendicular axis passing through the end on the floor and about which the stick undergoes rotational motion as it falls is

$$I = \frac{1}{3} ml^2.$$

Let ω be the angular speed of the free end of the stick as it hits the floor. Energy of the stick will be the rotational kinetic energy

$$E = \frac{1}{2} I\omega^2 = \frac{1}{6} ml^2\omega^2.$$

Speed v of the free end of the stick and its angular speed ω are related as



$$\omega = \frac{v}{l}.$$

From energy conservation, we have the relation

$$\frac{1}{6} mv^2 = \frac{mgl}{2},$$

or

$$v = \sqrt{3gl} = \sqrt{3 \times 9.8 \times 1.27} \text{ m s}^{-1} = 6.1 \text{ m s}^{-1}.$$