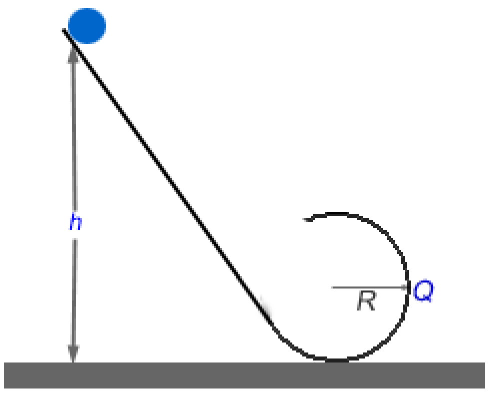
35.

Problem 12.15P (HRW) / 12.29 (HR)

We are given that a small solid marble of mass m and radius r will roll without slipping along the loop-the-loop track if it is released from rest somewhere on the straight section of track. We have to find (a) The minimum height h above the bottom of the track from where the marble must be released to ensure that it does not leave the track at the top of the loop (the radius of the loop-the-loop is R); (b) the horizontal component of the force acting on the marble at the point Q when it is released from height 6R above the bottom of the track. The following diagram illustrates the setup.



Solution:

We will solve this problem using the principle of conservation of energy. As the marble ball has a finite size and it rolls without slipping, magnitude of its rotational velocity will change with change in speed of its centre of mass. The speed of the ball at any instant when it is moving in the loop-the-loop can be found by equating at that instant the total kinetic energy of the ball with the change in the potential energy of the ball.

Let the speed of the centre of mass of the marble ball at some instant be v. At that instant its angular velocity ω will be $\frac{v}{r}$, as the radius of the marble ball is r and the ball is rolling without slipping. Moment of inertia, I, of a spherical ball of mass m and radius r is given by $I = \frac{2}{5}mr^2$. Therefore, the kinetic energy of the ball, when its speed is v, will be

$$K.E. = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2}{r^2},$$

or

$$K.E. = \tfrac{7}{10}mv^2.$$

As the ball is released when it is at rest, its $K.E.$ is equal to the change in its potential energy. When the change in the vertical height of the ball from its starting position is x, the change in potential energy

$$P.E. = mgx,$$

where g is the acceleration due to gravity. From conservation of energy, we get

$$v^2 = \tfrac{10}{7} mgx.$$

The other key concept that we will use in answering the problem is that when a body of mass m moves in a circular path of radius R with speed v, it is subjected to a centripetal force of magnitude mv^2/R. The direction of the centripetal force is along the line joining the body with the centre of the circular path, which changes as the body moves along its circular path.

a)

The minimum speed that the ball must have so that it does not leave the track when it is at the top of the loop-the-loop can be found by using the condition that at that location the centripetal force is provided by the weight of the ball. That is

$mv^2/R = mg,$

or

$v^2 = gR.$

Therefore, the minimum height h above the bottom of the track from which the ball has to be dropped so that it does not leave the top of the loop-the-loop will be given by the equation

$\frac{10}{7} g(h - 2R) = gR,$

or

$h = 2.7R.$

b)

The horizontal component of the force acting on the ball when it is at the point Q on the track will be the centripetal force. As the ball is released from a height $6R$ above the base of the track and the point Q is at a height R above the base of the loop-the-loop,

$v^2 = \frac{10}{7}(5R)g = \frac{50}{7} gR,$ and the horizontal component of the force at Q will be $mv^2/R = \frac{50}{7} mg.$