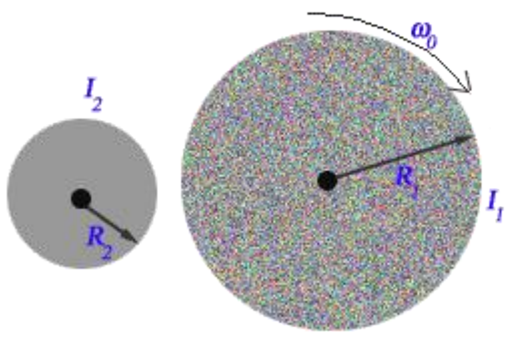
34.

Problem 12.49P (HRW)

Two cylinders having radii R_1 *and* R_2 *and rotational inertias* I_1 *and* I_2 *about the central axis are supported by axes perpendicular to the plane of the figure as shown. The large cylinder is initially rotating with angular velocity* ω_0*. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually, slipping ceases, and the two cylinders rotate at constant rates in opposite directions. We have to find the angular velocity* ω_2 *of the small cylinder in terms of* I_1, I_2, R_1, R_2*, and* ω_0*.*



Solution:

In this problem neither angular momentum nor kinetic energy are constants of motion. We will solve this problem by calculating changes in angular momentum arising out of angular impulse.

Let us assume that the two cylinders are in contact with each other for time Δt and let F be the magnitude of the force of friction exerted by each cylinder on the other when in contact and that at the end of this time interval both the cylinders begin to rotate without slipping. Let v be the speed of the rims of each cylinder after slipping has ceased. Let ω_2 be the angular speed of the smaller cylinder and ω_1 be the angular speed of the cylinder of radius R_1 when they begin to rotate without slipping. This implies

$$\omega_1 R_1 = \omega_2 R_2 \ .$$

Equations of angular impulse are

$$F\,R_1\,\Delta t = I_1\left(\omega_0 - \omega_1\right),$$

$$F\,R_2\,\Delta t = I_2\omega_2 \ .$$

Solving these three algebraic equations, we find

$$\omega_2 = \frac{I_1\omega_0}{\left(\dfrac{I_2R_1}{R_2} + \dfrac{I_1R_2}{R_1}\right)} \ .$$