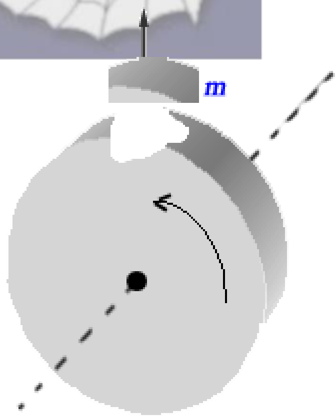
33.

<u>Problem 13.35 (HRK)</u>

A uniform disk of mass M and radius R rotates about a horizontal axis through its centre with angular speed ω_0. (a) *What is its kinetic energy? Its angular momentum?* (b) *a chip of mass m breaks off the edge at an instant such that the chip rises vertically above the point at which it broke off. How high above the point does it rise before starting to fall?* (c) *What is the final angular speed of the broken disk?*



<u>Solution:</u>

(a)

Rotational inertia of a flat disk of mass M and radius R about a horizontal axis passing through its centre is $\frac{1}{2}MR^2$.Kinetic energy of the disk will be

$$KE_i = \frac{1}{2}I\omega_0{}^2 = \frac{1}{4}MR^2\omega_0{}^2.$$

The angular momentum of the disk will be

$$L = I\omega_0 = \frac{1}{2}MR^2\omega_0{}^2.$$

(b)

As the chip of mass m shoots vertically up after it is chipped off the disk at its rim, its speed at that instant will be $\omega_0 R$.

It will go up by height h determined by the conservation of energy. That is

$$mgh = \frac{1}{2}m\omega_0{}^2R^2,$$

or

$$h = \frac{\omega_0{}^2R^2}{2g}.$$

Angular momentum of the chip with respect to the centre of the disk will be its value before it got chipped of the block and will remain constant during its motion. It is

$$L_{chip} = mR^2\omega_0.$$

(c)

Assuming that the piece of mass m is chipped off from the rim of the disk, the rotational inertia I' of the chipped disk will be

$$I' = \frac{1}{2}MR^2 - mR^2 = \left(\frac{M}{2} - m\right)R^2.$$

Let ω' be the angular speed of the chipped disk. From the conservation of angular momentum, we have

$$L = I'\omega' + m\omega_0 R^2,$$

or

$$\frac{1}{2}MR^2\omega_0 = \left(\frac{M}{2} - m\right)R^2\omega' + m\omega_0 R^2,$$

or

$$\omega' = \omega_0.$$