32.

Problem 13.27 (HR)

Two skaters, each of mass 50 kg, *approach each other along parallel paths separated by* 3.0 m. *They have equal and opposite velocities of* 10 m/s. *The first skater carries a long light pole,* 3.0 m *long, and the second skater grabs the end of it as he passes.* (*assume frictionless ice.*)
(a) *Describe quantitatively the motion of the skaters after they are connected by the pole.* (b) *By pulling on the pole, the skaters reduce their distance apart to* 1.0 m. *what is their motion then?* (c) *Compare the kinetic energy of the system in parts* (a) and (b). *Where does the change come from?*

Solution:

In a system in which the net external force and external torque is zero, the total linear momentum and the total angular momentum are constants of motion. As the two skaters have the same mass and are approaching each other with the same speed the linear momentum of this system is zero and will continue to remain zero as long as the skaters influence their motion by exerting internal forces on each other. That is their centre of mass will remain at rest.

We will find the angular momentum about a point midway between the two skaters. As the two skaters are moving horizontally the angular momentum will be in the direction of the vertical.

Angular momentum L of the two skaters as they approach each other will be

$L = 2\times 10\times 50\times 1.5 \text{ kg m}^2 \text{ s} = 1500 \text{ kg m}^2 \text{ s}.$

Rotational inertia of the system comprising of the two skaters is

$I = 2\times(50\times 1.5^2) \text{ kg m}^2 = 225 \text{ kg m}^2.$

(a)

At the instant when the two skaters get connected by holding the ends of a 3 m massless pole, motion of both of them towards each other will stop. But as they have a net angular momentum before they exert equal and opposite force on each other through the pole, which are internal to the system and so there is no external torque, and for the angular momentum to remain constant they will spin in the clockwise direction with angular velocity ω,

$$I\omega = L,$$

or

$$\omega = \frac{1500}{225}\text{ rad s}^{-1} = 6.7\text{ rad s}^{-1}.$$

(b)

After the skaters have reduced their separation to 1.0 m their rotational inertia will change to

$$I' = 2\times 50\times 0.5^2\text{ kg m}^2 = 25\text{ kg m}^2.$$

As the two skaters manage to reduce their rotational inertia so they will spin faster as angular momentum remains unchanged. Therefore, the changed angular speed with which the skaters will be spinning after they have come closer to each other will be

$$I'\omega' = L.$$

This gives

$$\omega' = \frac{1500}{25} \text{ rad s}^{-1} = 60 \text{ rad s}^{-1}.$$

(c)

Kinetic energy of the system in part (a) is

$$KE_a = \frac{L^2}{2I} = \frac{1500^2}{2 \times 225} \text{ J} = 5.0 \times 10^3 \text{ J},$$

and the kinetic energy of the system in part (b) is

$$KE_b = \frac{L^2}{2I'} = \frac{1500^2}{2 \times 25} \text{ J} = 45 \times 10^3 \text{ J}.$$

The additional kinetic energy comes from the work that each skater does against Coriolis forces when they come towards each other.