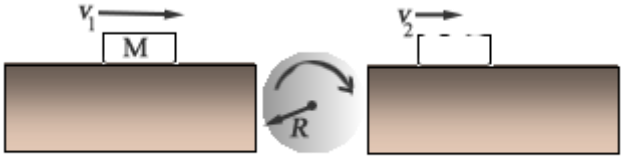
31.

Problem 13.21(HR)

The axis of the cylinder shown in the figure is fixed. The cylinder is initially at rest. The block of mass M is initially moving to the right without friction with speed v_1. *It passes over the cylinder to the dotted position. When it first makes contact with the cylinder, it slips on the cylinder, but the friction is large enough so that slipping ceases before M loses contact with the cylinder. The cylinder has a radius R and a rotational inertia I. We have to find the final speed* v_2 *in terms of* v_1, M, I and R.



Solution:

As there is no external torque acting on the system of the block and the cylinder, angular momentum about the axis of rotation of the cylinder is a constant of motion. We

will, therefore, use conservation of angular momentum for answering this problem.

We note that as initially the cylinder is at rest its angular momentum is zero. We will, therefore, first find the angular momentum of the block, moving to the right with velocity v_1, about the axis of rotation passing through the centre of the cylinder. Angular momentum of the block is $l_i = mv_1R$.

As the cylinder begins to rotate because of force of friction when the moving block comes into contact with it and the force of friction is so strong that slipping ceases when the block passes over the cylinder. Rotational speed ω of the cylinder and linear speed v_2 are related to each other, $v_2 = \omega R$.

Angular momentum of the block and the cylinder after the block has crossed over to the other side, as shown in the figure, will be

$$l_f = Mv_2R + I\omega,$$

or

$$l_f = Mv_2R\left(1 + I/MR^2\right).$$

Equating l_i and l_f, we get

$$v_2 = \frac{v_1}{1 + I/MR^2} \ .$$