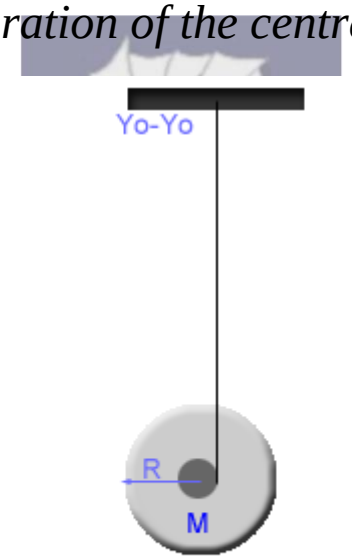
30.

Problem 19.17 (Fey)

A yo-yo like spool consists of two uniform disks, each of mass $M/2$ and radius R, and an axle of radius r and negligible mass. A thread wound around the axle is attached to the ceiling and the spool is released from rest a distance below the ceiling. We have to find the downward acceleration of the centre of the spool.

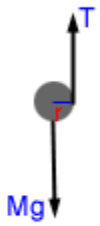


Solution:

Moment of inertia of each disk is $\frac{1}{2}MR^2$ and as the axle has negligible mass, the moment of inertia of the yo-yo will be

$I = MR^2$.

Axle of the Yo-yo disks rolls down on release without sliding. Therefore, there are two types of accelerations of the yo-yo, the translational acceleration of the centre of mass, a, and the angular acceleration, α, of the yo-yo about its axis. Application of the Newton's second law of motion and its counterpart for rotational motion gives two algebraic equations. Let T be the tension in the string. The free-body diagram of the yo-yo is



Free-body diagram of the axle of the yo-yo

From the free-body diagram we note that the net force acting vertically downwards is $mg - T$ and the torque, Tr, which results in angular acceleration in the anticlockwise direction. We get two equations of motion

$mg - T = Ma$,

and

$Tr = I\alpha$.

Also,

$a = \alpha R,$ and $I = MR^2$.

By eliminating T, we get

$$a = \frac{g}{1 + R^2 / r^2}.$$