29.

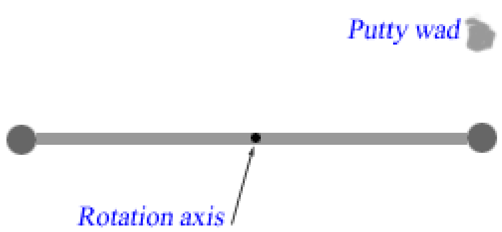
Problem 12.70P (HRW)

Two 2.00 kg *balls are attached to the ends of a thin rod of negligible mass,* 50.0 cm *long. The rod is free to rotate in a vertical plane without friction about a horizontal axis through its centre. While the rod is horizontal (see figure), a* 50.0 g *putty wad drops onto one of the balls with a speed of* 3.0 m/s *and sticks to it.*

(a) *What is the angular speed of the system just after the putty wad hits?*

(b) *What is the ratio of the kinetic energy of the entire system after the collision to that of the putty wad just before?*

(c) *Through what angle will the system rotate until it momentarily stops?*



Solution:

(a)

The key concept to be used in solving this problem is the principle of conservation of angular momentum.

The angular momentum of the putty wad, L, just before it hits the ball on the right about its centre of rotation as shown in the figure is

$$mvl\ (m = 0.05\ \text{kg}, v = 3.0\ \text{m s}^{-1}, l = 0.25\ \text{m}) = 0.0375\ \text{kg m}^2\text{s}^{-1}.$$

As the putty wad sticks to the ball on the right, the moment of inertia of the system comprising of two balls, a rod of negligible mass and the putty ball about its axis of rotation will be

$$I = \left(2.0 + 2.0 + 0.05\right) \times .25^2\,\text{kg m}^2 = 0.253\ \text{kg m}^2.$$

Let ω be the angular speed of the system just after the putty wad hits. By applying conservation of angular momentum, we get

$$L = I\omega = 0.0375\ \text{kg m}^2\text{s}^{-1}.$$

This gives

$$\omega = 0.148\ \text{rad s}^{-1}.$$

(b)

The *K.E.* of the putty wad just before collision

$\frac{1}{2}\times 0.05\times 3.0^2 \text{ J} = 0.225 \text{ J}.$

The *K.E.* of the entire system just after the putty wad hits can be computed by considering it as a rigid body. It is given by

$$\frac{L^2}{2I} = \frac{0.0375^2}{2\times 0.253}\text{J} = 2.78\times 10^{-3}\text{J}.$$

We thus find

$$\frac{K.E_{after\ collision}}{K.E._{before\ collision}} = \frac{2.78\times 10^{-3}}{0.225} = 0.0123.$$

(c)

When the system stops momentarily its kinetic energy would have completely changed into potential energy. Let θ be the angle by which the system would have rotated clockwise when it momentarily stops. The change in potential energy will be

$$(2.0 - 2.05)g\times 0.25\times \sin\theta = -0.123\sin\theta \text{ J}.$$

By equating the above value for the change in *P.E.* to $K.E._{after\ collision}$, we can calculate θ. We find

$$\theta = \sin^{-1}\left(-\frac{2.78\times 10^{-3}}{0.123} \right) = 181.3^{0}.$$