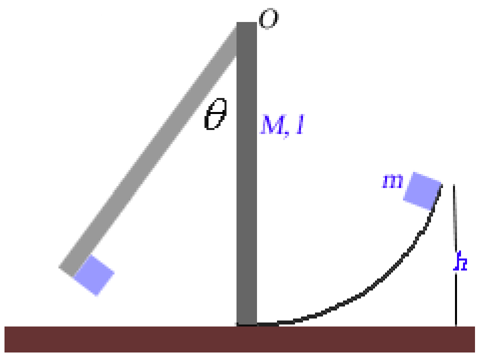
28.

Problem 12.69P(HRW)

The particle of mass m as shown in the figure slides down the frictionless surface and collides with a uniform vertical rod, sticking to it. The rod pivots about O through the angle θ before momentarily coming to rest. We have to find θ in terms of other parameters given in the figure.



Solution:

We will first find the distance, h_{cm}, of the centre of mass of the combined system of rod of mass M and the particle of mass m from the point *O*, when the mass m has just stuck to the rod. From the definition of the centre of mass, we get

$$h_{cm} = \frac{\frac{1}{2}Ml + ml}{M + m}.$$

The speed v of the particle just before its impact with the rod can be calculated from the principle of conservation of energy. We have

$$\tfrac{1}{2}mv^2 = mgh,$$

or

$$v = \sqrt{2gh}\ .$$

The angular momentum l of the particle about O is

$$l = mv = ml\sqrt{2gh}\ .$$

The moment of inertia I of the combined system of the rod and the particle about the axis passing through O is

$$I = \tfrac{1}{3}Ml^2 + ml^2,$$

or

$$I = l^2\left(\tfrac{1}{3}M + m)\right)\ .$$

Let ω be the angular speed of the rod and the particle just after the impact. From the principle of conservation of angular momentum, we have

$I\omega = ml\sqrt{2gh}$, which gives

$$\omega = \frac{ml\sqrt{2gh}}{I}\ .$$

The kinetic energy of the combined system of the rod and the particle just after the impact will be

$$K.E. = \tfrac{1}{2} I\omega^2 = \frac{m^2 l^2 gh}{I} = \frac{m^2 gh}{\left(\tfrac{1}{3}M + m\right)} \ .$$

We are given that the rod pivots about O through the angle θ before momentarily coming to rest. At that position the centre of mass of the rod and the particle will have risen in height by $h'_{cm} = \left(1 - \cos\theta\right) h_{cm}$. From the principle of conservation of energy we have

$$\tfrac{1}{2} I\omega^2 = \left(M + m\right) g h'_{cm},$$

or

$$\frac{m^2 gh}{\left(\tfrac{1}{3}M + m\right)} = \frac{\left(M + m\right)\left(\tfrac{1}{2}M + m\right) l \left(1 - \cos\theta\right)}{\left(M + m\right)},$$

or

$$1 - \cos\theta = \frac{m^2 h}{\left(\tfrac{1}{3}M + m\right)\left(\tfrac{1}{2}M + m\right) l},$$

or

$$\theta = \cos^{-1}\left[1 - \frac{6m^2 h}{l\left(2m + M\right))\left(3m + M\right)}\right] \ .$$