27.

Problem 12.62P (HRW)

A cockroach of mass m runs counterclockwise around a circular disk mounted on a vertical axis of radius R and rotational inertia I and having frictionless bearings. The cockroach's speed (relative to Earth) is v, whereas the disk turns clockwise with angular speed ω_0. *The cockroach finds a breadcrumb on the rim and of course stops.*

(a) *What is the angular speed of the disk after the cockroach stops?*

(b) *Is mechanical energy conserved?*

Solution:

(a)

We will apply the conservation of angular momentum for solving this problem. The angular momentum of the cockroach-disk system when the cockroach is moving with speed *v* in counterclockwise direction and the disk is turning with angular speed ω_0 will be

$$L = I\omega_0 - mvR.$$

After the cockroach stops the system of disk-cockroach will rotate together as a rigid body. Let ω be the changed angular speed of the system. As the moment of inertia of the disk-cockroach system is $I + mR^2$, the conservation of angular momentum implies

$$\left(I + mR^2\right)\omega = I\omega_0 - mvR,$$

or

$$\omega = \frac{I\omega_0 - mvR}{I + mR^2} .$$

(b)

The initial kinetic energy is $\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega_0^2\right)$ and the final kinetic energy is $\frac{L^2}{2\left(I + mR^2\right)} = \frac{\left(I\omega_0 - mvR\right)^2}{2\left(I + mR^2\right)}$. Therefore, the kinetic energy is not conserved.