26.

Problem 12.60P (HRW)

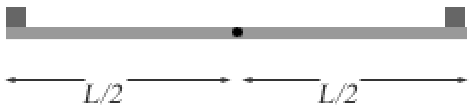
Two children, each with mass, sit on opposite ends of a narrow board with length L and mass M (the same as the mass of each child). The board is pivoted at its centre and is free to rotate in a horizontal circle without friction. (Treat is as a thin rod).

(a) *What is the rotational inertia of the board plus the children about a vertical axis through the centre of the board?*

(b) *What is the angular momentum of the system if it is rotating with angular speed* ω_0 *in a clockwise direction as seen from above? What is the direction of the angular momentum?*

(c) *While the system is rotating, the children pull themselves toward the centre of the board until they are half as far from the centre as before. What is the resulting angular speed in terms of* ω_0*?*

(d) *What is the change in the kinetic energy of the system as a result of the children changing their positions? (What is the source of the added kinetic energy?)*



Solution:

(a)

As the moment of inertia of a thin rod of length L and mass M about a axis perpendicular to it and passing through its centre is $\frac{1}{12}ML^2$, the total rotational inertia, I ,of the board and the children sitting as shown in the figure will be

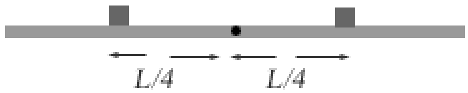
$$I = \tfrac{1}{12}ML^2 + 2 \times M \times \left(L/2\right)^2 = \tfrac{7}{12}ML^2 .$$

(b)

The angular momentum of the system as the children and the board are rotating with angular speed ω_0 in a clockwise direction as seen from above will be

$L = I\omega_0 = \tfrac{7}{12}ML^2\omega_0$, and it points downward.

(c)



The rotational inertia, I', of the system when the children have changed positions as shown in the figure will be

$$I' = \tfrac{1}{12}ML^2 + 2\times M \times (L/4)^2 = \tfrac{5}{24}ML^2.$$

Let ω' be the changed angular speed of the system. As the angular momentum of the system remains unchanged when there is no external torque acting on the system, we have

$I'\omega' = I\omega_0$. This gives

$$\omega' = \tfrac{14}{5}\omega_0.$$

The kinetic energy of a system rotating with rotational inertia I and angular momentum L is $L^2/2I$. Change in kinetic energy of the system will be

$$\frac{L^2}{2}\left(\frac{1}{I'} - \frac{1}{I}\right) = \frac{21}{40}ML^2\omega_0^2.$$

The kinetic energy is added to the system as children move inward and do work against Coriolis force acting on them.