24.

Problem 12.55E (HRW)

Suppose that the Sun run out of nuclear fuel and suddenly collapses to form a white dwarf star, with a diameter equal to that of the Earth. Assuming no mass loss, what would be then the Sun's new rotation speed, which currently is about 25 days? Assume that the Sun and the White dwarf are uniform, solid spheres.

Solution:

We note down the following astronomical data:

Mass of the Sun $= 1.99 \times 10^{30}$ kg.

Mean radius of the Sun, $R_{sun} = 6.96 \times 10^{8}$ m.

Mean radius of the Earth, $R_{earth} = 6.37 \times 10^{6}$ m.

The moment of inertia of a sphere of mass M and radius R,

$I = \frac{2}{5} MR^2$.

Angular speed of the Sun,

$$\omega_0 = \frac{2\pi}{25 \text{ day}} \times \frac{1 \text{ day}}{8.640 \times 10^4 \text{ s}} = 2.91 \times 10^{-6} \text{ rad/s} .$$

Angular momentum of the Sun $\frac{2}{5}MR_{sun}^2\omega_0$. Let ω be the angular speed of the Sun when it has become a white dwarf of radius R_{earth}. As there is no external torque on the Sun when it collapses, its angular momentum will be a constant of motion. Therefore,

$$I'\omega = I\omega_0.$$

This gives

$$\omega = \left(\frac{R_{sun}}{R_{earth}}\right)^2 \omega_0 = \left(\frac{6.96\times10^8}{6.37\times10^6}\right)^2 \times 2.91\times10^{-6} \ \text{rad}/\text{s}$$

$$= 3.47\times10^{-2} \ \text{rad}/\text{s}$$

Let the period of rotation of the white dwarf be *T*.

As $T = \dfrac{2\pi}{\omega}$, we find the rotational period of the Sun when it has become a white dwarf of radius R_{earth},

$$T = \frac{2\pi}{3.47\times10^{-2}} = 181.0 \text{ s} \ ; \ 3 \text{ min}.$$