Problem 12.47P (HRW)

Wheels A and B as shown in the figure are connected by a belt that does not slip. The radius of the wheel B is three times the radius of the wheel A. What would be the ratio of the rotational inertias I_A/I_B if both wheels had (a) the same angular momentum about their central axis and (b) the same rotational kinetic energies?



Solution:

Radius r_B of the wheel *B* is three times the radius r_A of the wheel *A*. That is $r_B = 3r_A$. As the two wheels are connected by a belt which does not slip, the speeds of the rims of the wheels will be equal and their angular speeds will be related as

$$\omega_{B}r_{B} = \omega_{A}r_{A},$$
and
$$\omega_{B} = \frac{1}{3}\omega_{A}.$$
(a)

If both wheels have the same angular momentum

$$I_B \omega_B = I_A \omega_A,$$

and

$$\frac{I_A}{I_B} = \frac{\omega_B}{\omega_A} = \frac{1}{3}.$$
(b)

If both wheels have the same rotational kinetic energy

$$\frac{1}{2}I_A\omega_A^2 = \frac{1}{2}I_B\omega_B^2,$$

and

$$\frac{I_A}{I_B} = \frac{1}{9}.$$

