22.

Problem 12.43 E (HRW)

The angular momentum of a flywheel having a rotational inertia of 0.140 kg m^2 *about its axis decreases from* 3.00 *to* 0.800 kg m^2/s *in* 1.50 s.

(a) *What is the average torque acting on the flywheel about its central axis during this period?*

(b) *Assuming a uniform acceleration, through what angle the flywheel will have turned?*

(c) *How much work was done on the wheel?*

(d) *What is the average power of the flywheel?*

Solution:

(a)

Equation of motion for rotation gives that change in angular momentum per second is the average torque acting on the flywheel. The change in angular momentum of the flywheel in 1.50 s is (0.800-3.00) kg m^2/s. Therefore, the average torque acting on the flywheel during this period will be

$$-\frac{2.2}{1.5}\ \text{kg m}^2\text{s}^{-2} = -1.46\ \text{N m.}$$

(b)

Let α be the average angular acceleration. The equation of motion is $I\alpha = \tau$, where I is the moment of inertia and α is the average acceleration. Therefore,

$$\alpha = \frac{-1.46\ \text{kg m}^2\text{s}^{-2}}{0.140\ \text{kg m}^2} = -10.43\ \text{rad s}^{-2}.$$

For rotational motion with constant acceleration the solution of the equation of motion is

$$\omega^2 - \omega_0^2 = 2\alpha\theta,$$

or

$$I^2\omega^2 - I^2\omega_0^2 = 2\alpha\theta I^2,$$

or

$$\theta = \frac{L_f^2 - L_i^2}{2\alpha I^2}\ .$$

L_f and L_i are the initial and final angular momentum, respectively.

Therefore,

$$\theta = \frac{0.8^2 - 3.0^2}{2 \times (-10.43) \times 0.140^2} = 20.45\ \text{rad}\ .$$

(c)

Work done on the flywheel, W, is equal to the change in its kinetic energy during the period. Therefore,

$$W = \frac{L_f^2 - L_i^2}{2I} = \frac{0.64 - 9.00}{2 \times 0.140}\,\text{J} = -29.9\ \text{J}.$$

(d)

The average power delivered by the flywheel is change in kinetic energy per second;

$$\text{Power} = \frac{29.9}{1.5}\ \text{W} = 19.9\ \text{W}.$$