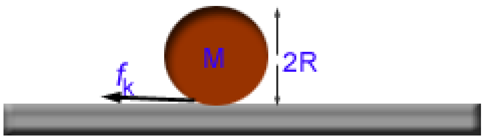
19.

<u>Problem 12.35 (HR)</u>

A bowling ball is thrown down the alley in such a way that it slides with a speed v_0 initially without rolling. We have to prove that it will roll without any sliding when its speed falls to $\frac{5}{7}v_0$. The transition from pure sliding to pure rolling is gradual, so that both sliding and *rolling takes place during this interval.*



<u>Solution:</u>

Let us assume that the mass of the ball is M and its radius is R. The ball on its release begins to slide in the direction in which it has been thrown. Therefore, it will experience a frictional force that will oppose its motion.

As the ball is sliding, the frictional force will be kinetic and let it be f_k and its direction will be as shown in the figure. The other forces that act on the ball are the weight

Mg acting at its centre of mass in the vertical direction and the normal force acting in the vertical direction at the point of contact of the ball with the surface of the alley way. The frictional force f_k will decelerate the sliding motion and it will also exert a torque about the centre of mass of the ball that will produce angular acceleration in the ball about an axis passing through the centre of the ball and perpendicular to the plane of the diagram.

The weight Mg and the normal force will not contribute to torque as their moment arm is zero. When the ball is viewed in its centre of mass frame it will appear to rotate with angular velocity that will be changing with time as long as the ball experiences the torque, $f_k R$. A stage will come in the motion of the ball when the velocity of the point of contact of the ball with the surface will become zero. At this stage the force of friction will cease and the ball will no longer slide and rotate at the same time and will start rolling without slipping.

We will now set up the equation of motion that determines the sliding speed of the ball and its angular speed as function of time. Let a be the magnitude of the sliding deceleration. It will be given by the Newton's second law of motion

$$a = \frac{f_k}{M}.$$

The speed with which the ball will be sliding at time t after it has been released with speed v_0 is given by the equation of motion of a decelerating body

$$v = v_0 - \frac{f_k}{M} t.$$

The moment of inertia of a spherical ball of radius R and mass M is $\frac{2}{5} MR^2$. The angular acceleration α of the ball is related to the torque $f_k R$ and the moment of inertia by the counterpart of the Newton's second law of motion for rotational motion

$$\frac{2}{5} MR^2 \alpha = f_k R,$$

or

$$\alpha = \frac{5}{2} \frac{f_k}{MR}.$$

The angular speed ω as a function of time will be αt. Or

$\omega = \frac{5}{2}\frac{f_k t}{MR}$. The speed of the point of contact of the rotating ball with the surface as measured from its centre of mass will be $\omega R = \frac{5}{2}\frac{f_k t}{M}$. The ball will stop sliding and begin to roll without slipping at time t', when

$$v_0 - \frac{f_k t'}{M} = \frac{5}{2}\frac{f_k t'}{M},$$

or

$$t' = \frac{2}{7}\frac{M v_0}{f_k}$$

The reduced sliding speed at t' when the ball will stop sliding and begin rolling without slipping will be

$$v' = v_0 - at' = \frac{5}{7}v_{0}.$$