17.

Problem 12.11P (HRW)

A body of radius R and mass m is rolling smoothly with speed v on a horizontal surface. It then rolls up a hill to a maximum height h.

(a) *If* $h = 3v^2/4g$, *what is the body's rotational inertia about the rotational axis through its centre of mass?*

(b) What might the body be?

Solution:

As the body of radius R is rolling smoothly, its angular speed ω and speed of its centre of mass are related by the formula $\omega = v/R$. Let the rotational inertia of the body about its axis of rotation passing through the centre of mass be I. The kinetic energy of the body will be the sum of the kinetic energy of translation of its centre of mass, $\frac{1}{2}mv^2$, and the kinetic energy of rotation about the axis passing through its centre of mass $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(Iv^2/R^2\right)$. That is

$$\text{K.E.}=\frac{1}{2}v^2\left(m+\frac{I}{R^2}\right)$$

When the body rolls up a hill to the maximum height $h=3v^2/4g$, it will possess only potential energy, which will be

$$\text{P.E.}=mgh=\frac{3mv^2}{4g}.$$

The principle of conservation of energy gives us the equation

$$\frac{1}{2}v^2\left(m+\frac{I}{R^2}\right)=\frac{3mv^2}{4g}.$$

Solving this equation, we get

$$I=\frac{1}{2}mR^2.$$

Therefore, the body is either a solid cylinder or a disk.