

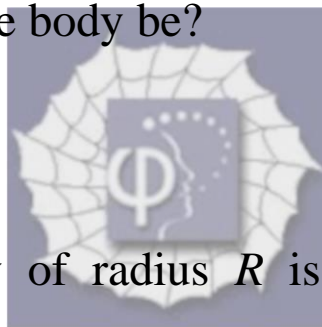
17.

Problem 12.11P (HRW)

A body of radius R and mass m is rolling smoothly with speed v on a horizontal surface. It then rolls up a hill to a maximum height h .

(a) If $h = 3v^2/4g$, what is the body's rotational inertia about the rotational axis through its centre of mass?

(b) What might the body be?



Solution:

As the body of radius R is rolling smoothly, its angular speed ω and speed of its centre of mass are related by the formula $\omega = v/R$. Let the rotational inertia of the body about its axis of rotation passing through the centre of mass be I . The kinetic energy of the body will be the sum of the kinetic energy of translation of its centre of mass, $\frac{1}{2}mv^2$, and the kinetic energy of rotation about the axis passing through its centre of mass

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{Iv^2}{R^2}\right). \text{ That is}$$

$$\text{K.E.} = \frac{1}{2} v^2 \left(m + \frac{I}{R^2} \right)$$

When the body rolls up a hill to the maximum height $h = 3v^2/4g$, it will possess only potential energy, which will be

$$\text{P.E.} = mgh = \frac{3mv^2}{4g}.$$

The principle of conservation of energy gives us the equation

$$\frac{1}{2} v^2 \left(m + \frac{I}{R^2} \right) = \frac{3mv^2}{4g}.$$

Solving this equation, we get

$$I = \frac{1}{2} mR^2.$$

Therefore, the body is either a solid cylinder or a disk.

