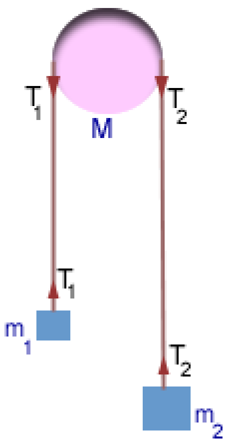
15.

<u>Problem 11.94 (HRW)</u>

We are given that two blocks, mass m_1 (400g) *and mass* m_2 (600g), *are connected by a massless cord that is wrapped around a disk of mass* M (500g) *and radius* R (12cm). *The disk can rotate without friction about fixed horizontal axis through its centre. The cord cannot slip on the disk. We have to find the magnitude of the acceleration of the blocks, the tension* T_1 *in the cord at the left and the tension* T_2 *in the cord at the right.*



SOLUTION:

We will solve this problem by using the Newton's laws for translational and rotational motions. As the cord does not slip and the disk rotates because of torque on it, the relation between the linear acceleration of the blocks, a, and the angular acceleration, α, of the disk is $a = \alpha R$. We will draw free-body diagrams for the blocks and that for the disk and apply the laws of motion for setting up the equations of motion connecting tensions T_1 and T_2, masses of the blocks etc.

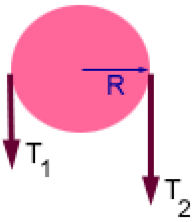
We will analyse this problem further and demonstrate conservation of energy. We will compute the change in potential energy of the system as the masses move from and show that at any instant the change in the potential energy is equal to the sum of kinetic energy of the two masses and the rotational energy of the disk.

As $m_2 > m_1$ and the two masses are connected by an unstretchable massless cord, the magnitude of acceleration a of the two masses will be equal. The mass m_2 will accelerate downward and the mass m_1 will accelerate upward. Let I be the moment of inertia of the pulley. As it is a disk of mass M and radius, R, $I = MR^2$.

From the free-body diagrams of masses m_1 and m_2 by applying Newton's second law of motion we get

$$T_1 - m_1 g = m_1 a$$

$$m_2 g - T_2 = m_2 a.$$



Free-body diagram of the disk

The third equation of motion is obtained by equating the net torque on the disk, $T_2R - T_1R$, to $I\alpha$, the rotational counterpart of the Newton's second law of motion. It is

$$(T_2 - T_1)R = I\alpha.$$

These three equations can be algebraically solved for finding expressions for a, T_1 and T_2 in terms of m_1, m_2, M and the acceleration due to gravity g.

The solutions are

$$a = \frac{(m_2 - m_1)g}{(m_1 + m_2 + M/2)},$$

$$T_1 = m_1 g(2m_2 + M/2)/(m_1 + m_2 + M/2),$$

$$T_2 = m_2 g(2m_1 + M/2)/(m_1 + m_2 + M/2).$$

Substituting $m_1 = 0.4kg,\ m_2 = 0.6kg,\ M = 0.5kg$, we find
$a = 1.57ms^{-2}$, $T_1 = 4.55N$ and $T_2 = 4.94N$.

We will now verify the law of conservation of energy. Let us observe the system at a time when the block m_2 has fallen by height h beginning from rest. At that instant the block m_1 would have risen by height h. Let the speed of the two blocks at that instant be v. As the blocks move with constant acceleration, $v^2 = 2ah$. Therefore, the total kinetic energy of the two blocks will be $\frac{1}{2}(m_1 + m_2)v^2 = (m_1 + m_2)ah.$

The angular velocity ω of the pulley at that instant will be v/R.

Therefore, the rotational energy of the pulley will be $\tfrac{1}{2} I\omega^2 = \tfrac{1}{4} Mv^2$.

The total motion energy of the blocks and the pulley will be the sum of these two expressions and is $(m_1 + m_2)ah + \tfrac{1}{4} Mv^2 = (m_1 + m_2 + M/2)ah$.

Substituting the expression for a that we have found by solving the equations of motion, we find that at that instant the total motion energy of the system is $(m_2 - m_1)gh$, which is the change in the potential energy of the two blocks.

Therefore, the change in potential energy of the system is equal to the change in its motion energy. This is the law of conservation of energy.