

14.

Problem 11.88P (HRW)

Attached to each end of a thin steel rod of length 1.20 m and mass 6.40 kg is a small ball of mass 1.06 kg. The rod is constrained to rotate in a horizontal plane about a vertical axis through its midpoint. At a certain instant, it is observed to be rotating with an angular velocity of 39.0 rev/s. Because of friction, it comes to rest 32.0 s later. Assuming a constant frictional torque, we have to compute (a) the angular acceleration, (b) the retarding torque exerted by the friction, (c) the total mechanical energy dissipated by the friction, and (d) the number of revolutions exerted during the 32.0 s. (e) Now suppose that the frictional torque is known not to be constant. Which, if any, of the quantities (a), (b), (c) or (d) can still be computed without additional information? If such a quantity exists, we have to give its value.

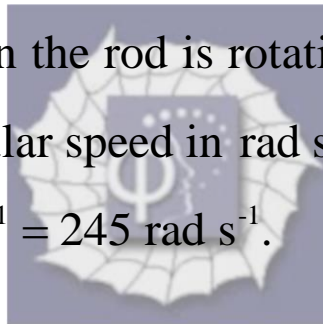
Solution:

Let l be the length of steel rod and M (6.40 kg) be its mass. Let m (1.06 kg) be the mass of balls attached to each end of the rod at a distance $l/2$ from the centre of the rod. The rotational inertia of the system about axis perpendicular to the rod and passing through its centre is

$$\begin{aligned} I &= \frac{1}{12} Ml^2 + 2 \times m \left(\frac{l}{2} \right)^2, \\ &= \left(\frac{1}{12} \times 6.4 + \frac{1}{2} \times 1.06 \right) 1.20^2 \text{ kg m}^2, \\ &= 1.53 \text{ kg m}^2. \end{aligned}$$

At an instant when the rod is rotating with angular speed 39.0 rev/s its angular speed in rad s^{-1} is

$$\omega = 39 \times 2\pi \text{ rad s}^{-1} = 245 \text{ rad s}^{-1}.$$



(a)

It is given that due to friction rod comes to a stop from the instant when its angular speed was 245 rad s^{-1} in 32 s. If we assume that the torque exerted by friction on the rod was constant, the acceleration of the rod during this period was

$$\alpha = -\frac{245}{32} \text{ rad s}^{-2} = -7.66 \text{ rad s}^{-2}.$$

(b)

Retarding torque exerted by friction on the system is given by

$$\tau = I\alpha = -1.53 \times 7.66 \text{ kg m}^2\text{s}^{-2} = 11.71 \text{ kg m}^2\text{s}^{-2}.$$

(c)

$$\begin{aligned}\text{Total energy dissipated by the system} &= \frac{1}{2} I \omega^2 \\ &= \frac{1}{2} \times 1.53 \times (245)^2 \text{ J} \\ &= 4.6 \times 10^4 \text{ J}.\end{aligned}$$



(d)

Number of revolutions executed by the system during 32 s can be calculated from the relation

$$\omega^2 = 2\alpha\theta.$$

We calculate θ ,

$$\begin{aligned}\theta &= \frac{(245)^2}{2 \times 7.66} \text{ rad} = 3918 \text{ rad}, \\ &= \frac{3918}{2\pi} \text{ rev} = 624 \text{ rev}.\end{aligned}$$

(e)

When the frictional force is not known to be constant, from the given data, we can calculate only the amount of energy dissipated by the system.

