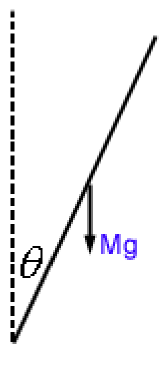
13.

<u>Problem 11.87P (HRW)</u>

A tall, cylinder-shaped chimney falls over when its base is ruptured. Treating the chimney as a thin rod with height h, express the (a) *radial* (b) *tangential components of the linear acceleration of the top of the chimney as a function of the angle* θ *made by the chimney with the vertical.* (c) *At what angle* θ *does the linear acceleration equal g?*

<u>Solution:</u>



Let the mass of the chimney be M. When the chimney is standing vertically its weight Mg will act at its centre of mass, which is at a height $h/2$ from the base. At the instant when the chimney is at angle θ with the vertical, the drop in the height of the centre of gravity will be $\dfrac{h(1-\cos\theta)}{2}$. The change in the potential energy of the chimney will be $\frac{1}{2}Mgh\left(1-\cos\theta\right)$. The moment of inertia I of the chimney about the end at the ground is $\frac{1}{3}Mh^2$. Therefore, if we denote the angular speed of the chimney treated as a rigid body when it is inclined at an angle θ with the vertical as ω, the rotational energy of the chimney will be $\frac{1}{2}I\omega^2$. Equating the change in potential energy to the change in kinetic energy, we get

$\frac{1}{2}I\omega^2 = \frac{1}{2}Mgh(1-\cos\theta)$.

Substituting for I we find

$$\omega^2 = \frac{3g}{h}\left(1-\cos\theta\right).$$

The radial acceleration is the centripetal acceleration and is $\omega^2 h$. It will therefore be $3g\left(1-\cos\theta\right)$.

For finding the linear acceleration of the top end of the chimney we first calculate its angular acceleration α. The torque, τ, on the chimney about its rotational axis, when it is at an incline θ, will be $\frac{1}{2}hmg\sin\theta$. Using the relation

$I\alpha = \tau$,

we find

$\alpha = \frac{3}{2}\left(g\sin\theta/h\right)$.

The linear acceleration of the top end

$$\alpha h = \tfrac{3}{2} g \sin\theta .$$

The linear acceleration will be equal to *g*, at an angle θ given by the equation

$$\tfrac{3}{2} g \sin\theta = g,$$

or

$$\sin\theta = \tfrac{2}{3}, \text{ or}$$

$$\theta = 41.8^0.$$