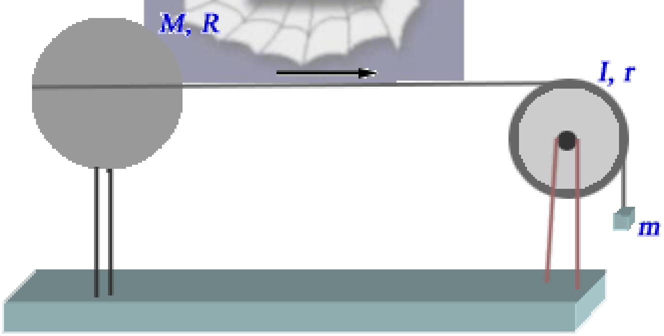
12.

Problem 11.86P (HRW)

A uniform spherical shell of mass M and radius R rotates bout a vertical axis on frictionless bearings. A massless cord passes around the equator of the shell, over a pulley of rotational inertia I and radius r, and is attached to a small object of mass m. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it has fallen a distance h from rest?



Solution:

Let v be the instantaneous speed of the object of mass m after it has fallen from rest by distance h. As the chord does not slip on the pulley and there is no loss of energy due to friction on the pulley's axis, the angular speed of the pulley will be

$$\omega_P = \frac{v}{r},$$

and the angular speed of the spherical shell will be

$$\omega_S = \frac{v}{R}.$$

The rotational inertia of the pulley is I and the rotational inertia of a thin shell of radius R and mass M is $\frac{2}{3}MR^2$.

The total kinetic energy of the system at the moment when the object has speed v will be the sum of the rotational energy of the shell plus the rotational energy of the pulley plus the kinetic energy of the object. This gives

$$K.E. = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} + \frac{1}{2}\times\frac{2}{3}MR^2\times\frac{v^2}{R^2},$$

$$= \frac{1}{2}v^2\left(m + \frac{I}{r^2} + \frac{2}{3}M\right).$$

The change in potential energy of the system is mgh.

Equating the change in kinetic energy to the change in potential energy, we get

$$\frac{1}{2}v^2\left(m + \frac{I}{r^2} + \frac{2}{3}M\right) = mgh.$$

This gives

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{r^2} + \frac{2}{3}M}} \quad .$$