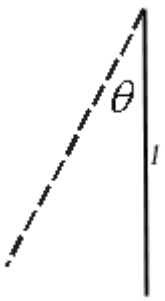
10.

Problem 11.80E(HRW)

A thin rod of length l and mass m is suspended freely from one end. It is pulled to one side and then allowed to swing like a pendulum, passing through its lowest position with an angular speed ω. (a) *Calculate its kinetic energy as it passes through its lowest position.* (b) *How high does its centre of mass rise above its lowest position? Neglect friction and air resistance.*

Solution:



(a) Let the mass of the rod of length l be m. Its moment of inertia, I, about the axis perpendicular to its plane of oscillation and passing through its upper end will be $\frac{1}{3}ml^2$. As the angular speed of the swinging rod at its lowest position is ω, the kinetic energy of the rod as it passes through its lowest position will be

$\frac{1}{2}I\omega^2 = \frac{1}{6}ml^2\omega^2$.

(b)

Let θ be the maximum angle that the rod makes with the vertical during its swing. At this position the kinetic energy of the rod will be zero and its potential energy will be determined by the change in height, Δh_{cm}, of the centre of mass from its lowest position. $\Delta h_{cm} = \frac{1}{2} l - l\cos\theta$. By equating the change in potential energy to change in kinetic energy, we can find, Δh_{cm},

$\Delta h_{cm}\, mg = \frac{1}{6} ml^2\omega^2$

or

$\Delta h_{cm} = \frac{1}{6} l^2\omega^2 / g.$