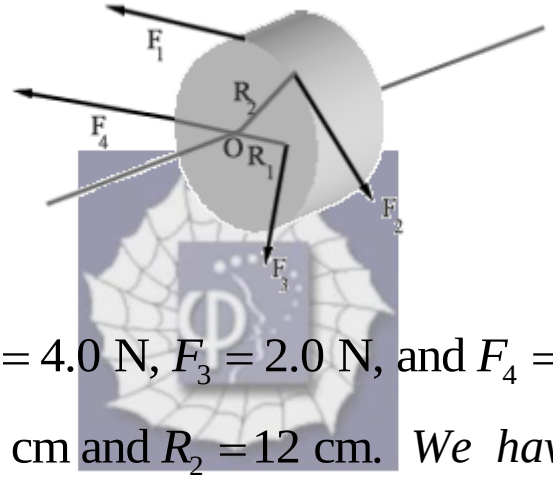
9.

Problem 11.67E (HRW)

A cylinder having a mass of 2.0 kg *can rotate about its central axis through point* O. *Forces are applied as shown in the figure:*



$F_1 = 6.0$ N, $F_2 = 4.0$ N, $F_3 = 2.0$ N, and $F_4 = 5.0$ N.

Also, $R_1 = 5.0$ cm and $R_2 = 12$ cm. *We have to find the magnitude and direction of the angular acceleration of the cylinder.* (*During their rotation, the forces maintain their same angles relative to the cylinder*).

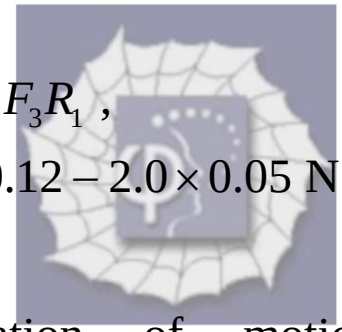
Solution:

Mass of the cylinder *m* is 2.0 kg and its radius R_2 is 0.12 m. Rotational inertia of the cylinder about its central axis

$$I = \frac{1}{2} m R_2^{\,2} = \frac{1}{2} \times 2.0 \times 0.12^2 \text{ kg m}^2 = 0.0144 \text{ kg m}^2.$$

Torque on the cylinder by each force is its magnitude times its moment arm and its direction is as determined by the cross-product rule. We will call the torque in the anticlockwise direction about the central axis positive. From the diagram we note that the angles between the forces and the line joining the point O to their point of application are either 90^0 or 0^0. Therefore, the net torque is

$$\begin{aligned}\tau &= F_1 R_2 - F_2 R_2 - F_3 R_1 \,, \\ &= \left(6.0 - 4.0\right) \times 0.12 - 2.0 \times 0.05 \text{ N m}, \\ &= 0.14 \text{ N m}.\end{aligned}$$



Rotational equation of motion relates angular acceleration α and torque on a rigid body

$$\alpha I = \tau.$$

Therefore, the angular acceleration of the cylinder will be

$$\alpha = \frac{0.14}{0.0144} \text{ rad s}^{-2} = 9.7 \text{ rad s}^{-2}.$$