Problem 11.50E (HRW)

Two particles, each with mass m, are fastened to each other, and to a rotation axis at O, by two thin rods, each with length l and mass M as shown in the figure. The combination rotates around the rotation axis with angular velocity ω . Obtain algebraic expressions for (a) the rotational inertia of the combination about O and (b) the kinetic energy of rotation about O.



Solution:

(a)

As mass of each rod is *m* and their combined length is 2*l*, the moment of inertia of the rods with respect to rotation axis at *O* is $\frac{1}{3} \times 2M \times (2l)^2 = \frac{8}{3}Ml^2$.

The moment of inertia of the two balls about *O* is $ml^2 + m(2l)^2 = 5ml^2$.

Therefore, the algebraic expression for the rotational inertia of the combination about O is

$$I=\frac{8}{3}Ml^2+5ml^2.$$

(b)

And, the kinetic energy of rotation about O is

$$KE_{rot} = \frac{1}{2} \times I\omega^2 = \frac{1}{2} \left(\frac{8}{3}M + 5m\right) l^2 \omega^2.$$