4.

Problem 11.46P (HRW)

The oxygen molecule, O_2*, has a total mass of* 5.30×10^{-26} kg *and a rotational inertia of* 1.94×10^{-46} kg m^2 *about an axis through the centre of the line joining the atoms and perpendicular to that line. Suppose that such a molecule in a gas has a speed of* 500 m s^{-1} *and that its rotational kinetic energy is two-thirds of its translational kinetic energy. We have to find its angular velocity.*

Solution:

Mass of the oxygen molecule, $M_{O_2} = 5.30\times10^{-26}$ kg.

Its rotational inertia, $I = 1.94\times10^{-46}$ kg m^2.

Speed of the molecule, $v = 500$ m s^{-1}.

Its kinetic energy of translation,

$$KE_{tr} = \tfrac{1}{2}M_{o_2}v^2 = \frac{1}{2}\times5.30\times10^{-26}\times500^2 \text{ J} = 6.625\times10^{-21} \text{ J.}$$

In the problem it has been given that the rotational kinetic energy of the molecule is two-thirds of its translational kinetic energy. That is

$$KE_{rot} = \frac{2}{3} \times 6.625 \times 10^{-21}\ \text{J} = 4.416 \times 10^{-21}\ \text{J}.$$

Rotational velocity, ω, of the molecule can be calculated from the relation

$$KE_{ro} = \tfrac{1}{2} I\omega^2.$$

Therefore,

$$\omega = \sqrt{2KE_{ro}/I} = \sqrt{2 \times 4.416 \times 10^{-21} / 1.94 \times 10^{-46}}$$

$$= 6.7 \times 10^{12}\ \text{rad s}^{-1}.$$