3.

<u>Problem 11.44P (HRW)</u>

A pulsar is a rapidly rotating neutron star that emits radio pulses with precise synchronisation, one such pulse for each rotation of the star. The ***period*** *T of rotation is found by measuring the time between pulses. At present, the pulsar in the central region of the Crab nebula has a period of rotation of* T=o.033 s, *and this period is observed to be increasing at the rate of* 1.26×10^{-5} s/y.

(a) *What is the value of the angular acceleration in* rad/s^2?

(b) *If its angular acceleration is constant, how many years from now will the pulsar stop rotating?*

(c) *The pulsar originated in a supernova explosion seen in the year* A.D. 1054. *What was T for the pulsar when it was born?* (Assume constant angular acceleration since then.)

<u>Solution:</u>

(a)

Angular speed ω and period T are related as

$$\omega = \frac{2\pi}{T}.$$

Therefore,

$$\frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}.$$

We have been given that for the pulsar

$$\frac{dT}{dt} = 1.26\times10^{-5}\ \text{s/y} = \frac{1.26\times10^{-5}}{3.156\times10^{7}} = 3.99\times10^{-13}.$$

Therefore, the angular acceleration of the pulsar is

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{0.033^2}\times3.99\times10^{-13}\ \text{rad/s}^2 = -2.30\times10^{-9}\ \text{rad/s}^2.$$

(b)

The present angular speed of the pulsar is

$$\omega = \frac{2\pi}{0.033}\ \text{rad s}^{-1} = 1.90\times10^{2}\ \text{rad s}^{-1}.$$

If the pulsar will continue to decelerate at this rate, it will stop rotating at time t_f,

$$t_f = \frac{\omega}{\alpha} = \frac{1.90\times10^{2}}{2.30\times10^{-9}} \text{ s} = 8.27\times10^{10} \text{ s},$$

or

$$t_f = \frac{8.27\times10^{10}}{3.156\times10^{7}} \text{ y} = 2621 \text{ y}.$$

(c)

As the pulsar was formed in 1054 A.D. its life till the present has been 949 y. Its period of rotation at the time of its birth

$$T_i = (0.033 - 1.26\times10^{-5}\times949) \text{ s} = 0.021 \text{ s}.$$